

Turbo Tessellate Version 1.1

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Introduction

The purpose of this function is to perform tessellations of the once-punctured torus (regular and flared) and display the results on the Poincaré unit disc model. Turbo Tessellate uses a depth-first recursion method that applies only those transformations that will provide visible results. This allows the user to create highly detailed images in a short amount of time, and even tessellate in real-time as the input variables are changed.

How to Use this Package

Place this file in the same directory as your project notebook. You can then load the package easily by inserting the following code at the beginning of your notebook:

```
SetDirectory[NotebookDirectory[]];  
  
<< TurboTessellate_v1.1.m
```

Then go to Cell → Cell Properties and click on "Initialization Cell". This will ensure the package will be loaded when you evaluate your notebook.

Tessellating Hyperbolic Space with Ideal Quadrilaterals

- **Input**

The required input for a basic tessellation is:

```
TurboTessellate[A, B]
```

Where A and B are two matrices that form a subgroup of $\text{PSL}(2, \mathbb{R})$. When the commutator of A and B is parabolic, a complete tiling of the hyperbolic plane will occur. When the commutator is not parabolic, an incomplete tiling of the hyperbolic plane will occur.

- **Threshold Parameters**

TurboTessellate allows for two additional parameters, Threshold and MaxWordLength. Threshold sets the minimum size of the tessellation, while MaxWordLength specifies the maximum depth of the tessellation.

When using a specified Threshold and MaxWordLength, the required input is:

```
TurboTessellate[A, B, Threshold, MaxWordLength]
```

The minimum recommended value for Threshold is .01. The range for MaxWordLength is 1 to 100. By default these parameters are set to .1 and 50, respectively.

Tessellating Hyperbolic Space with Fundamental Domains that are not Ideal Quadrilaterals

When the commutator of A and B is hyperbolic, tessellating hyperbolic space with an ideal quadrilateral will result in an incomplete tiling. However, when the commutator is hyperbolic, a complete tiling can be achieved by using a region in hyperbolic space bounded by four disjoint sides with eight distinct endpoints on the boundary at infinity. Such a region is not compact and will meet the boundary at infinity in four arcs. This region will be a fundamental domain for a once-punctured torus with a flared end (i.e. infinite area).

- **Input**

In this case, TurboTessellate requires an additional input: FirstPoint. Each value of FirstPoint (range -1 to 1) corresponds to a different tiling of the flared torus for the same marked group, with the values -1 and 1 corresponding to incomplete tilings.

The required input for this tessellation is:

```
TurboTessellate[A, B, FirstPoint]
```

- **Threshold Parameters**

Threshold and MaxWordLength can also be set using this method. The required input is:

```
TurboTessellate[A, B, FirstPoint, Threshold, MaxWordLength]
```

The minimum value for Threshold is .01. The range for MaxWordLength is 1 to 100. By default these parameters are set to .1 and 50, respectively.

Optimizing TurboTessellate Based on Desired Output

- **Dynamic Rendering**

The rendering of a tessellation can be a computationally intensive task in *Mathematica*. However, the method used by TurboTessellate allows for detailed tessellations to be drawn in real time using the Dynamic or Manipulate commands.

When using TurboTessellate to dynamically render a tessellation, keep in mind the total number of objects drawn will limit the performance. For this reason, it is generally recommended to set the Threshold parameter above .2 and MaxWordLength below 25 when using TurboTessellate dynamically.

- **Static Renders**

To create a highly detailed tessellation, set Threshold to .01 and MaxWordLength to 100.

- **Adjusting Colors and Image Size**

TurboTessellate has two additional parameters to allow the user to configure the colors of the geodesics, as well as the resulting image size. ColorList determines the values relating to *Mathematica*'s RGBColor graphics directive, while Size determines the pixel dimensions of the resulting graphics object. Both parameters must be specified at the same time along with Threshold and MaxWordLength, such as:

```
TurboTessellate[A, B, Threshold, MaxWordLength, ColorList, Size]
```

Or:

```
TurboTessellate[A, B, FirstPoint, Threshold, MaxWordLength, ColorList,  
Size]
```

ColorList should be formatted in the following manner: {{R1,G1,B1},{R2,G2,B2},{R3,G3,B3}}. Size should be an integer. "DefaultColors" can be substituted for ColorList if the user only wishes to change the image size.